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ELEMENTARY PARTICLE STRUCTURE

CHAPTER II

ELECTROMAGNETIC FIELD

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There are several ways in which a new theory may be presented. Each way has its own special advantages. The logical-axiomatic formulation is of special value in eliminating internal inconsistencies in a theory, and in establishing a rigorous basis for the prediction of physical phenomena. Chapter I was primarily concerned with the extraction of certain mathematical consequences from the fundamental postulate, that there exists a single primitive wave field out of which all the elementary particles are built. In Chapter III this logical-axiomatic point of view will be extended to the fundamental phenomena of gravitation.

A new theory can also be presented in a more descriptive fashion. That is, the concepts and procedures can be compared to other existing theories and pictures of nature.

Points of contact and points of departure can be established or suggested. The current theories have been shown to describe nature very accurately, within certain realms of applicability, and to encounter obstacles in other realms. A new and inclusive theory ought to account for their successes and failures, and to redress the latter. It is of high importance that a theory be presented in such a way that its ideas are really communicated. A new language, freshly defined, may be needed in the logical-axiomatic formulation, but for purposes of communication an older language with familiar symbols and pictures is preferable.

A new theory can also be presented in a third way, with emphasis on the motivation and the train of thought leading to its principal concepts. Such a chronological or historical presentation has a particular value of its own. There are ordinarily many forks in the path, many unfruitful sidetracks and unsuccessful shortcuts. A careful discussion of the alternatives to the way that was chosen may serve to turn up a new concept which shortens the route to the destination.

THE GRAVITATIONAL SEA

The results of Chapter I, from the deductive or postulational or logical-axiomatic point of view, might be summarized in the form of a postulate and a theorem:

<u>Postulate</u>: All of the elementary particles are built from a single primitive wave field.

<u>Theorem I</u>: The equations for the simplest possible multiple-wave structure (a double-wave system of zero internal angular momentum) have solutions in which a parameter, κ, with the dimension of reciprocal length or wavenumber, appears in an essential way. The wavenumber κ determines the inner periodicity and the scale of size of the structure, both for the particle-like solution and for the zero-frequency solution; and also serves as the unit of mass for the particle-like solution.

In Chapter III the postulational approach will be continued and applied to the phenomena of gravitation. Arguments based on the postulate above will be used in introducing a second theorem:

<u>Theorem II</u>: The gravitational field is represented by a Fermi sea of primitive waves. The top of the Fermi sea is marked by a wavenumber κ, which varies slowly through space, rising higher where there is a greater concentration of elementary particles. The Fermi sea provides a boundary condition for the structures representing the elementary particles, since these structures must join the sea at its surface. That is, of the primitive waves forming the structures, those waves which are not localized to the interior of the elementary particles must have the periodicity κ of the surface of the gravitational Fermi sea.

The double-wave solutions in Theorem I are of such a form that they satisfy the boundary condition in Theorem II, with the same value for the wavenumber, κ, being used in the two theorems. It will be a purpose of later chapters to generalize Theorem I to more complicated multiple-wave structures, as partial proof of a third theorem:

<u>Theorem III</u>: The wavenumber κ, which marks the surface of the gravitational Fermi sea, determines thereby the scale of size of all the elementary particles and of any combination of particles; and serves as the unit of mass for the elementary particle mass spectrum.

The reciprocal of the wavenumber κ thus serves as a locally valid "fundamental length," although it is not an absolute constant but a measure of the gravitational potential. In Chapter III it is shown that the gravitational acceleration of an elementary particle nearly at rest can be represented by the gradient of a potential:

$$\vec{g} = \nabla(c^2 \log_e \kappa) \qquad (1)$$

But it is also shown that when the particle is in motion through space there is an additional effect, due to the refraction of the primitive waves in the sloping Fermi sea, and that this additional effect accounts quantitatively for the bending of light and the other "crucial phenomena" of gravitation. However, in both the acceleration (1) and the refraction it is only

through the gradient of its logarithm that the wavenumber κ enters the gravitational equations. Thus the numerical value of κ cannot be determined from gravitational calculations. Nevertheless, from the identification of a structure in Chapter IV as one of the elementary particles of finite mass, it will be concluded that the elementary particle mass spectrum can be accounted for, most satisfactorily, by the assignment:

$$\frac{1}{\kappa} = 2.8 \cdot 10^{-13} \text{ cm} \qquad (2)$$

The study of other structures, as in Chapter V, will be used to reinforce the assignment (2), and to increase its precision.

It might seem a logical conflict in terms, for a numerical value to be assigned to a quantity which is supposed to vary through space. There is really no inconsistency, however. If (2) is considered to be a local assignment at the earth's surface, then the variation needed to account for gravitational phenomena is so gradual that κ will be greater at the surface of the sun by only two parts per million.

Furthermore, according to Theorem III any ruler will itself change in size in being transported between two points at different gravitational potentials, since a ruler is a combination of elementary particles. If in Equation (2) the "centimeter" is transformed in this way into a "local centimeter", then the numerical factor in (2) becomes an absolute ratio, a ratio between two lengths which vary in the same proportion through space. If, on the other hand, the "centimeter" in (2) is taken to be the length of a standard centimeter block which is kept on the earth, then a ruler transported to the sun will actually shorten in length. If such a ruler were used to measure a diameter of the earth's orbit, and also the circumference of a circle about this diameter, then the ratio of the circumference to the diameter would be found to be less than π. Yet if the measurements were corrected to allow for the shortening of the ruler near the sun, the corrected ratio of circumference to diameter would equal π exactly, according to the above picture in which gravitational effects are attributed to a field in space rather than to any modification of space itself.

COMPARISON WITH GENERAL RELATIVITY

How does the above picture of gravitation compare with that given by general relativity? In Einstein's general theory of relativity, space itself accounts for gravitational phenomena. The motion of an elementary particle is along a "geodesic" in space, the observed accelerations being ascribed to the curvature or non-Euclidean character of space in the neighborhood of a gravitating mass. There is a physical argument which suggests, but does not prove, the validity of such a picture of gravitation. The line of reasoning starts from the observation that all masses, all elementary particles or combinations of particles, are accelerated in the same way by a gravitational force. It follows that, in a frame of reference which is carried along by one particle, falling freely, a second particle, also falling freely, will appear to have no acceleration. This is the "equivalence principle," stating that a gravitational force is equivalent to an acceleration of coordinates, for all elementary particles. From the equivalence principle it is argued that gravitational effects should be ascribed to the properties of space itself, and space is then found to be non-Euclidean.

However, a somewhat different line of reasoning could also be pursued, and from the same original observation, that all elementary particles respond in the same way to a gravitational force. This observation does not require that gravitation be a property of space. Gravitational effects can quite as easily be attributed to a field in space, if it can be shown that all of the elementary particles are so closely related to one another that they will respond to the gravitational field in the same way. Partly this is a semantic problem, since one can simply define the gravitational field as that part of the general field which treats all elementary particles on the same footing. But it is still necessary to show that there exists a part of the general field which does treat them all the same, and which accounts quantitatively for the known gravitational phenomena. With such a demonstration, as summarized in Theorem II and given in detail in Chapter III, it becomes permissible to treat the gravitational field as a field occupying space rather than as a modification of space itself.

Once the gravitational field has been set up as an entity distinguished from the space it occupies, the observer is free to choose, for his description of space, any system of coordinates which he finds to be convenient. He may choose a system which is at rest, with respect to himself, or one which is in relative motion with a constant velocity; the descriptions of the same fields or phenomena, as referred to these two systems of coordinates, are related through the familiar transformations of special relativity. The observer may also choose an accelerated system, if he uses the transformations of general relativity. And now, in addition, the observer may generalize one step further and choose coordinates which are either Euclidean or non-Euclidean, to suit his convenience. A transformation between coordinates with different curvatures must be accompanied by the appropriate transformation of the gravitational field, a transformation so chosen that actual physical phenomena are not affected. Of particular interest will be the transformation between a Euclidean frame with a varying gravitational potential, as described in Chapter III, and that particular non-Euclidean frame which leaves the gravitational potential a constant.

It is a natural generalization of Einstein's arguments to require that the theory be made invariant to transformations between Euclidean and non-Euclidean coordinate frames. From an operational point of view, the systems of coordinates by means of which an observer correlates his observations are not themselves measurable. Only the fields or particles in space can be observed, and the choice of coordinates can have no physical consequence, since any two systems of coordinates encompassing the same physical phenomena can in practice be mapped, one upon the other. For example, in general relativity the Schwarzschild solution for the "line element" in the vicinity of a point source of gravitation is actually expressed in terms of a "laboratory system" of polar coordinates in Euclidean space, and the calculation of a planetary orbit proceeds in the laboratory frame. If the "line element" is interpreted simply as a measure of a particular kind of field occupying Euclidean space, the equations of that field being the equations of general relativity, then only the words are changed, not the experimental consequences of the theory. As a matter of fact, Einstein's own picture of a field in space transcends the simpler interpretation of general relativity which is sometimes given. In his words, "The field of a

material particle may the less be viewed as a _pure gravitational field_ the closer one comes to the position of the particle. If one had the field-equation of the total field, one would be compelled to demand that the particles themselves would _everywhere_ be describable as singularity-free solutions of the completed field-equations. Only then would the general theory of relativity be a _complete_ theory." It is difficult to interpret these words in terms of a field which is simply a modification of the coordinate system, while the interpretation in terms of a separate and occasionally very complicated field, occupying ordinary space, is readily made.

The generalization of the field concept, to include electromagnetism as well as gravitation, is the subject of the present chapter. This introductory summary of the gravitational parts of the theory has been necessary because the electromagnetic field, in this theory, is a spin-dependent modification of that same Fermi sea whose height measures the gravitational potential. The present theory is consistent with Einstein's concept of a "total field" which describes the elementary particles and which also provides the locally effective mechanism by which one particle responds to the influence of all the others.

FIELDS AND OPERATORS

If a mathematical description of the "total field" is to be provided, then this description should include rules for selecting from this total field those special fields, such as the gravitational and electromagnetic fields, which are measured in particular experiments. The procedure to be used is the familiar quantum-mechanical formalism. The total field is given by a wave function satisfying a system of linear differential equations, while the special fields are represented by operators. Measurable field quantities are obtained from expectation values and transition matrix elements of the operators, taken over the wave functions which describe particular physical situations.

However, before the theory can be considered complete, it must be shown that the formalism described above can be derived from the fundamental postulate. A first step in the derivation is taken in Chapter III, where it is found that the equations describing the refraction of a primitive wave can be written in the spinor form of Chapter I, with the wave function

components given as in (I.8) and (I.9). However, it is found also that there are phase factors, written as arbitrary in (I.12), which are no longer entirely arbitrary in the spinor form of the gravitational equations in Chapter III. To be sure, the initial magnitude of the phase angle appears to be arbitrary, but when the wave is refracted the phase angle transforms like a multiple (plus or minus one-half) of a "third Eulerian angle" measuring rotations about the propagation vector $\vec{\kappa}$ as an axis. With the help of this third Eulerian angle it is found in Chapter III that the spinor wave function can be made single-valued with respect to the rotation-inversion group of transformations.

While this phase angle, described above, plays an important role in the causal progression of a primitive wave, the numerical value of the angle does not affect the quantities which are of significance for gravitational calculations. But it is possible to obtain, from a bilinear combination of the components of the spinor wave function ξ, those quantities κ and $\vec{\kappa}$ which are of gravitational significance; the undesired phase angle is eliminated through

the linking of components of ξ^* with components of ξ in the bilinear combination. Each component of ξ^* or ξ is the product of $\sqrt{\kappa}$ and one of the Cayley-Klein parameters, so that a bilinear combination has the dimension 1/L, the dimension of a wavenumber. Since the gravitational quantities κ and $\vec{\kappa}$ both have this same dimension 1/L, only a simple bilinear combination is needed, with a dimensionless operator to sort the components into the proper combinations. The usual notation for a hermitian scalar product will be used:

$$(\varphi, \psi) = \sum_i \varphi_i^* \psi_i \qquad (3)$$

(In the case that the wave functions φ and ψ are multiple-wave functions, the summation will be generalized to include integration over internal variables.) There may also be an integration over the "external" variables, in some cases, but such an integration will not ordinarily converge, in the simplest formulation of the theory, since the primitive waves are not "bound" in the usual sense. In the following expressions, which relate κ and $\vec{\kappa}$ to the operators unity and $\tau_\xi \vec{\sigma}$, respectively, no such external integration is implied.

$$\kappa = (\xi, \xi) \qquad (4a)$$

$$\vec{\kappa} = (\xi, \tau_\zeta \vec{\sigma} \xi) \qquad (4b)$$

In (4b) the τ-spin operator τ_ζ is the same as the operator called τ in Chapter I. It is one of a set of three operators $\tau_\xi, \tau_\eta, \tau_\zeta$ analogous to the three σ-spin operators $\sigma_x, \sigma_y, \sigma_z$ and having similar matrix representations to those given for $\vec{\sigma}$ in Chapter I, equations (I.3).

The results, from Chapter III, which have been described in the preceding two paragraphs, can be summarized in a fourth theorem:

Theorem IV: The equations for a gravitationally refracted primitive wave, which were obtained from the gravitational analysis in Theorem II, can be written in spinor form, in terms of the spinor components used in Theorem I. The spinor wave function has the dimension $1/\sqrt{L}$. The gravitational field is characterized by quantities having the dimension $1/L$. Each such quantity is expressed, in terms of the spinor wave function, as the expectation value of a dimensionless operator.

ROTATIONS

The spinor formulation also makes possible a simple field representation for rotations. An observer may choose to describe certain physical phenomena as they would be observed from a rotating coordinate system. According to the general field picture used in the present theory, the transformation from a fixed to a rotating set of coordinates should be accompanied by a transformation of the field, a transformation so chosen that physical phenomena are not affected.

In the absence of rotations or accelerations, that is, in an "inertial" or freely falling reference frame, the vacuum is represented by a Fermi sea with a flat surface. When viewed from an accelerated coordinate system, the same sea acquires a sloping surface, and the same kind of sloping surface is used to represent a gravitational force, in this theory. How can such a field representation be extended to include rotations?

The answer is found in the spin parameters which help to characterize a primitive wave. The spinor

wave function has four components, two choices of σ-spin and two choices of τ-spin. Thus there is an opportunity for correlation between the height of the sea and the spin components. In effect, there are four separate seas intermingled. The average, or gravitational sea, as given in (4a), may be modified by, or accompanied by, rotational effects which can be measured by an axial vector $\vec{s}$:

$$\vec{s} = (\xi, \vec{\sigma}\, \xi) \qquad (5)$$

The absence of rotational effects means the vanishing of $\vec{s}$ in the vacuum, at least on the average. The use of a rotating coordinate system means a non-vanishing value of $\vec{s}$. Dimensionally $\vec{s}$ is a wavenumber; when multiplied by the velocity c it becomes an angular velocity measuring the rate of rotation of the coordinate frame, with the direction of $\vec{s}$ marking the axis of rotation.

Any discussion of rotations raises a familiar question. If the observer is free to choose whatever coordinates he finds convenient, then who is to say which coordinates are fixed and which are rotating?

The reply, of course, is that all axes are equally valid; that in any particular frame the local effects of distant matter can be represented by a particular field distribution; and that in certain frames the field distribution takes a particularly simple form. For example, the rotational effects measured by (5) should be a minimum in a frame which makes the stars and galaxies appear to be at rest, although a rotating frame, with (5) not a minimum, may occasionally prove to be more convenient for the description of certain special phenomena.

In the present theory the "inertial" properties of an elementary particle are determined by the character of the Fermi sea in its vicinity, as given in (4a) and (5). The Fermi sea, in turn, is a summation over the elementary particles in the universe (with appropriate weighting factors). Thus the present theory is in harmony with "Mach's principle," which states that the inertial properties of one particle are a manifestation of its interaction with the rest of the matter in the universe. The identification of inertial mass and gravitational

mass also follows from the present theory, since there is no place for inertial properties except through gravitation. The gravitational Fermi sea provides the unit of mass for a particular particle and determines its local behavior. Inertial motion (rectilinear motion in an "inertial" or free-fall frame) is just a special case of gravitational response; it is motion in a gravitational sea which happens to be flat.

The inertial properties of a bit of matter cannot be considered as inherent in that bit of matter, but should be considered as induced by the total distribution of matter in the universe. From an operational point of view no intrinsic mass can be attributed to an isolated elementary particle in an otherwise empty universe. Additional particles are needed to determine a set of coordinate axes from which velocities and accelerations can be measured; otherwise the concept of inertia cannot be given a precise meaning. It is consistent with this operational argument that, if mass is a property induced by the gravitational field, then the unit of mass should become a measure of the gravitational potential, as it is in the present theory.

When the spin of the primitive waves is taken into account, as in (5), the Fermi sea is enriched in its properties and becomes capable of inducing inertia of rotation. If (5) varies through a region of space, as viewed by a particular observer, then a structure moving through that region will experience a torque, as seen by this same observer.

The interpretation of rotational inertia can be illustrated by a classic example. If the water in a pail is stirred until it rotates with a certain angular velocity, the surface of the water will become concave, the water rising at the edges of the pail above the level at the center. (A pailful of mercury, stirred at the same angular velocity, would acquire the same concavity, except for small corrections due to surface tension, etc.) Viewed from a coordinate system rotating at the same angular velocity as the liquid in the pail, the liquid itself will be at rest but the concavity will remain. According to Mach's principle, the apparent force, which in the rotating frame seems to be pulling the liquid to the sides of the pail, is to be attributed to the apparent motion, with respect to this same rotating

coordinate system, of the stars and galaxies which seem to be moving in circular orbits about a common axis. The field picture allows the influence of the distant motion to be translated into a local field distribution. In the present theory the field distribution is given in the form (5) at the center of rotation, with a more complicated variation of (5) and (4a) farther out. In Einstein's formulation the field distribution is given as a spatial dependence of the g_{ij}. In either case the problem is to show that the local field is determined by the location and motion of the distant masses. The field itself is not as simple, for rotations, as the field around a source of gravitation. The rotation field is velocity-dependent in a direction-sensitive way. Viewed, as before, from the rotating frame, the liquid in the pail will react to further stirring in a manner which depends upon the direction of the further stirring. If the further stirring is in the same direction as before, the concavity will increase, whereas if the direction is reversed, the liquid surface will tend to flatten. It is as if, seen in the rotating frame, the liquid has a kind of intrinsic angular momentum to start with, induced by the "spin" of the Fermi sea occupying the vacuum.

MULTIPLE-WAVE FIELDS

The field picture of rotations, described in the previous section, was brought into this second chapter not so much because of its practical value, which may be small, but rather because it serves as an introduction to the electromagnetic field, as it is represented in the present theory. And the electromagnetic field, in its turn, will help to introduce the fields of still greater complexity which are discussed in later chapters.

The electromagnetic field, in this theory, is a double-wave field. Like the rotation field, the electromagnetic field is a spin-dependent modification of the gravitational Fermi sea, but the modification is more complicated. For the rotation field the simple Fermi sea was enriched by a correlation between σ-spin and height of sea. For the electromagnetic field the sea will be further enriched by a cross-correlation between primitive waves considered in pairs, a particular kind of cross-correlation involving both τ-spin and σ-spin. The nature of the cross-correlation is such that a pair of waves is involved only if the two waves have opposite directions of τ-spin.

Questions immediately present themselves. What is the physical meaning of a double-wave distribution? How can the consideration of waves by pairs produce anything not already contained in the single-wave picture? The second question can be answered first. A complete characterization of all the primitive waves, one by one, will of course contain all details, including everything that could be included in a characterization by pairs. But the simple picture of a Fermi sea, even with the rotational σ-spin dependence included, does not represent a complete characterization, since the picture includes an implicit averaging process. There has been an implicit average over time, to smooth out or smear out any spatial correlations between waves, and (what is not entirely the same thing) there has been an average over phase angles, including the "third Eulerian angle" which has been mentioned earlier as being needed for a complete characterization of a primitive wave. According to Theorem IV, those properties of a single primitive wave which are needed for the gravitational (and rotational) field are obtained from expectation values of operators; in the computation of expectation values a necessary property of a single wave, its phase, is lost.

Actually, there are two phases which are lost. Of the four components of the spinor wave function ξ, two components have $\tau_\zeta = +1$, while the other two have $\tau_\zeta = -1$. The time variable enters into the phase factors of the four components always with the same sign, but the "third Eulerian angle" enters the phase factors with one algebraic sign for the first pair of components and with the opposite sign for the second pair. Since none of the single-wave operators contemplated in Theorem XV involve τ_ξ or τ_η (which would mix the two pairs of components), the expectation values lose both the sum of the phases (thus averaging over time) and the difference of the phases (thus averaging over the third Eulerian angle).

As long as a wave is characterized by "observables," or the expectation values of operators, the characterization cannot be complete since phase factors have been lost. If the spinor wave function itself is used, then the characterization is mathematically complete but contains phases which cannot be determined by experiment. However, these phases are actual and significant in the interaction of waves with each other, so that they should have secondary observable consequences when several waves interact.

In other words, there is plenty of opportunity for new observables to be formed when waves are considered by pairs, since the list of single-wave observables does not exhaust the degrees of freedom of a single wave. Later in this second chapter there will be exhibited a set of double-wave observables satisfying Maxwell's equations and providing a suitable representation of the electromagnetic vectors, charges, and currents. Similarly, additional observables, not contemplated in the double-wave characterization, are to be expected from the analysis of triple-wave systems in Chapters IV and V.

A complete description of the primitive field is included in the spinor formulation, but this spinor formulation contains unobservable phases. If the description is restricted to observables (expectation values of operators, with averages taken over phases, and with integrations and summations over internal space and spin variables), then any such description which undertakes to give a complete characterization may have to include not only single-wave operators, but also double-wave operators, triple-wave operators,

and operators of higher multiplicities, perhaps without limit. It is the program of later chapters to examine some of the multiplicities in turn, but the mathematical and philosophical problem of the possible existence of a limit will also be examined, since it is related not only to the number of possible kinds of elementary particles but also to the general problem of causality, and the meaning of measurement. The epistemological implications of the theory will be discussed shortly.

Two questions were asked, several paragraphs back: What is the physical meaning of a double-wave distribution? How can the consideration of waves by pairs produce anything not already contained in the single-wave picture? The second question was answered through a discussion of the distinction between the representation of the primitive field by its spinor wave function and the representation by a list of observables. The former picture is complete but not observable, while the latter picture, though observable, cannot approach completeness without the inclusion of double-wave observables, and may still be incomplete even after the inclusion of any finite list of multiple-wave observables.

And now the first of the two questions can be considered. What is the physical meaning of a double-wave distribution? How can it be measured? What is its operational meaning? How can two waves be observed as a single effect? For example, the electric vector is represented in this theory by a double-wave operator; an electric field distribution is a correlation of waves in the Fermi sea, a double-wave correlation involving both σ-spin and τ-spin, a correlation between waves having opposite signs for τ_ζ, so that $\tau_{1\zeta} + \tau_{2\zeta} = 0$. In practice and in principle, an electric field is measured through the introduction of a test charge and the observation of its response to the field. In the present theory, the simplest test charge is an electron (or a charged mu-meson), which is a structure built from three primitive waves, two having one sign for τ_ζ, the third having the other sign for τ_ζ. (The explicit structure is discussed in Chapter V.) It should be clear that, when a triple-wave structure is used as the test particle, a response to a double-wave distribution is just as easy to account for as a response to a single-wave distribution. It should also be clear that the analogous neutral

structure (neutrino and neutral mu-meson, described in Chapter IV), built from three primitive waves all with the same sign for τ_ξ, will not respond to the double-wave distribution representing the electric field strength. In order to respond to a double-wave structure with "anticorrelation" of τ-spins, a triple-wave structure must contain a pair of waves with opposite τ-spin, whereas the neutral structure of Chapter IV does not contain such a pair. In fact, it is through the lack of response to an electric field that the triple-wave structure of Chapter IV is found to be neutral, so that the two solutions (one of zero mass and one of finite mass) can be tentatively identified as the neutrino and the neutral mu-meson.

In general, a multiple-wave field distribution is a modification of the Fermi sea which can only be described in terms of correlations among several waves taken as a unit, the number of waves giving the multiplicity. In practice and principle, the multiple-wave distribution is measured through the observation of the response of an elementary particle which is built from at least as many primitive waves as are correlated in the multiple-wave distribution.

DIMENSIONAL ARGUMENTS

A further generalization can be made concerning multiple-wave fields. In Theorem IV the dimension of a single-wave spinor function was given as $1/\sqrt{L}$, or $L^{-1/2}$. Accordingly a double-wave function will have the dimension L^{-1}, and a triple-wave function the dimension $L^{-3/2}$. An expectation value is bilinear in the wave function, so that the dimension of the expectation value of a dimensionless operator will just be the square of the dimension of the wave function. Operators which have original dimensions of their own, as operators, will have these dimensions augmented when expectation values are formed.

This theory expresses all physical quantities in very simple dimensions, using only length, time and number, with the velocity c, the velocity of a primitive wave (and of massless structures like the photon and neutrino), being used to relate the scale of length to the scale of time. In terms of these simple units, the dimensions of many physical quantities can readily be written down, from the usual theories, and this makes it easier to identify the operators, in the present theory, which represent those physical quantities.

In the single-wave case, as has been pointed out, the pertinent observables, given in (4) and (5), have the dimension of L^{-1}, or wavenumber. The observable (5) is more appropriately expressed as a frequency, after multiplication by the velocity c. The operators in (4) and (5) are dimensionless, the dimension L^{-1} being the square of the dimension of the single-wave function.

In the case of a double-wave function, the expectation value of a dimensionless operator will have the dimension L^{-2}, or "per unit area." The natural units for the electric and magnetic field vectors, in terms of length, time, and number, would be "lines per unit area," where the "lines" are "lines of force", a numerical rather than a dimensional concept. Thus the dimension of the electric vector, from the point of view of the present theory, will be L^{-2}. A comparison with the square of the dimension of the double-wave function shows that the electric vector should be represented by a dimensionless operator in this theory, and a similar comparison shows that the magnetic vector should likewise be represented by a dimensionless operator.

The number of vector operators which can be built from the available dimensionless double-wave spin operators is severely limited. In fact, there are only three independent vector combinations of σ-spin operators, $(\vec{\sigma}_1 + \vec{\sigma}_2)$, $(\vec{\sigma}_1 - \vec{\sigma}_2)$, and $(\vec{\sigma}_1 \times \vec{\sigma}_2)$. With appropriate τ-spin factors added, two of these operators are found to play the roles of electric and magnetic vectors in a set of Maxwell's equations, with other operators, not dimensionless, appearing in the equations as charge and current operators. The details are given later in this chapter. As was to be expected, the operator for electric charge contains a derivative and thus has the operator dimension L^{-1}, so that the dimension of the expectation value is L^{-3}, the appropriate dimension for "charge density."

Dimensional arguments can also be applied to the triple-wave case. Here the expectation value of a dimensionless operator will be L^{-3}, or "per unit volume." The wave function itself has the dimension $L^{-3/2}$. In Chapter V is discussed a triple-wave structure to represent the electron, so that it is the existing theory of the electron which should be compared with the triple-wave field, in order to establish an identification. The existing theory of the electron is quantum mechanics,

in which the electron is represented by a wave function which is a "probability amplitude" having the dimension $L^{-3/2}$. When its absolute value is squared, the result is a "probability density" having the dimension L^{-3}. The direct identification of the quantum-mechanical wave function with the triple-wave function of the present theory is suggested by the dimensional analysis.

However, in quantum mechanics it is ordinarily assumed that the probability density for the electron is the same thing (except for a factor) as the charge density distribution. In the present theory there is a fundamental distinction between the two densities. The probability density is a triple-wave property referring to the probable location of the center of an electron; while the charge density is a double-wave property and can, for example, refer to the distribution of charge within an electron, since in this theory the charge of an electron is not concentrated at its central point. In the newer quantum electrodynamics the charge associated with an electron is no longer concentrated at a point, but is distributed in a way which corresponds more closely with the picture of the electron as a triple-wave structure, as given in the present theory.

It has been implied above that quantum mechanics, with its probability-amplitude interpretation of the wave function, is a special theory applicable only to triple-wave structures (electron, neutrino, charged and neutral mu-mesons). Certain qualifications need to be appended to this interpretation. Certain features of quantum mechanics will be found to recur in the description of other elementary particles. Later in this chapter an explicit wave function for the photon will be described. It satisfies a wave equation resembling an equation from quantum mechanics, but the wave function cannot be interpreted as a probability amplitude because, by Theorem IV, it has the wrong dimension. The actual wave equation is linear and homogeneous in the wave function, so that the form of the equation does not restrict the dimension of its solution, but Theorem IV does provide such a restriction and rules out the interpretation of the photon wave function as a probability amplitude.

Within the framework of the present theory, the charged and neutral pi-mesons have tentatively been identified as structures built from four primitive waves. They have wave equations and wave functions, but again the wave functions cannot be interpreted as probability amplitudes because of Theorem IV.

However, for the proton and neutron, in this theory, there is a certain partial validity to the interpretation of the wave function as a probability amplitude. From arguments based on stability, spin, and mass, it is tentatively concluded that a nucleon is built from fifteen primitive waves. The observed stability is evidence of a "closed shell," in the building up of elementary particles from primitive waves. From the order of magnitude of the observed mass it is concluded that the shell which has been filled is a P-shell. There are four choices of spin for a primitive wave, counting both τ-spin and σ-spin, and three independent P-state angular functions, so that a closed P-shell will have twelve primitive waves. Twelve primitive waves will have a mass contribution which is at most 12κ, but may be less if there are correlation effects which reduce it. With the value (2) for κ, the mass of a nucleon is about 13.4κ, so that the nucleon can be interpreted as the combination of a closed P-shell and several more primitive waves. From the spin and statistics of the nucleon it is inferred that the total number of waves used must be odd. The number must be greater than 13,

because the mass of a 13-wave structure cannot exceed 13 κ, while the nucleon mass is 13.4 κ. (The upper limit of N κ for the mass of an N-wave structure is found to be a consequence of the boundary condition mentioned in Theorem II.) Thus the number of waves in a nucleon structure must be at least 15, though the above arguments do not rule out the number 17 or any higher odd number. However, strong evidence for the number 15 can be derived from the results of Chapter IV. In Chapter IV it is shown that an uncharged triple-wave structure exists with the mass 1.48 κ. This is interpreted as a neutral mu-meson, a particle whose existence has been surmised but not yet verified experimentally. A combination of a closed P-shell (mass 12 κ) with such a neutral triple-wave structure (mass 1.48 κ) might be expected to give a 15-wave uncharged structure with a mass somewhat less than 13.48 κ. perhaps 13.42 κ, which is the observed mass of the neutron in the units of this theory. A similar combination of a charged triple-wave structure from Chapter V (representing a charged mu-meson) with the same kind of closed P-shell should give a charged 15-wave structure of about the same mass, perhaps 13.40 κ, which is the mass of the proton on the same scale.

In this way, by a comparison of the observed nucleon masses with the approximate masses to be predicted for structures of 15 waves, and by a consideration of the observed spin, statistics, and stability, a tentative assignment of 15 waves is selected for the neutron and proton. Further support for this assignment can be derived from a dimensional argument. It is found that the equations of quantum mechanics, along with the interpretation of the wave function as a probability amplitude, apply quite well to neutrons and protons, as well as to electrons, and apply generally to nuclei and atoms and molecules, etc., all of which are combinations of neutrons, protons, and electrons. However, there must be introduced into the equations a short-range nuclear interaction or "meson field." All of this follows from the present theory as long as the 12-wave P-shell is identified with the "meson field," leaving the rest of the nucleon, the remaining triple-wave structure, to represent the nucleon in the equations, the effects of the P-shell being absorbed into the quantum-mechanical mass and the short-range coupling. A later chapter will examine the detailed consequences of such a picture of the nucleon.

CAUSALITY

The preceding pages of this second chapter have been intended as a comparatively nonmathematical and descriptive summary of the concepts underlying this elementary particle theory, particularly the concept of the gravitational Fermi sea and its modifications. Since in this theory all of the elementary particles are multiple-wave structures, it is of basic importance to make clear the meaning of the multiple-wave fields, and the distinction between the total field and the observable characteristics of that field. Particularly significant is the definition of "observability," the delineation of the limits of knowledge or control, the problem of causality within and outside those limits. Before the detailed equations of the electromagnetic field are introduced, a brief discussion of the moot problem of causality will be presented.

In elementary particle physics the word "causality" has come to be used in two senses, a restricted sense and a more general sense. In the general sense of the term, there is a long-standing controversy over the causal character of the laws of physics. The equations

of quantum mechanics, when directed to the prediction of experimental results, can give only statistical predictions concerning many physical quantities which can be measured to high accuracy. Similarly, and apart from whatever theory is used, it is found experimentally that, no matter how carefully the apparatus is designed or operated, certain experimental results are subject to an irreducible scatter or fluctuation, and can be controlled only to this extent: they can be kept within a certain statistical distribution. It is Einstein's contention that the laws of nature must be exact, so that there must be variables and equations which determine precisely where in the statistical distribution the result of a particular experiment will fall; and that eventually the experimenter will learn how to fix or select particular values of the extra variables and thus to determine the result of the experiment precisely. In disagreement with Einstein are others who consider nature, in certain aspects of its detailed behavior, to be unknowable and inscrutable. The present theory, in Theorem IV and the sections following it, takes an intermediate position in this controversy.

According to the present theory, the laws of nature are exact and determined and causal, but an essential part of the cause is internal to the object of experiment and is thus outside the control of the experimenter, outside his information but not outside his understanding, unknowable but not inscrutable. The experimenter cannot control the experiment completely and must repeat the experiment many times in order to be able to average over the inner variable or variables which were not under his control. To be useful to this experimenter, a theory must do the same thing, must average over internal variables and give statistical predictions.

The mathematical device of the expectation value, discussed in connection with Theorem IV, has for its main purpose the averaging over internal degrees of freedom, so that the "observables," which represent the incomplete description available to the experimenter, may be obtained from the wave function, where the wave function is the exact description which alone obeys the strict requirements of causality. But can it really be demonstrated that the experimenter's description is necessarily incomplete?

It is possible to give a line of argument, of a more philosophical tone than the derivation of Theorem IV, leading to the conclusion that there should be inner degrees of freedom which play an essential part in causal behavior but are not observable. As the first step in the argument, it is to be noted that the wave equation of the primitive field is a linear differential equation, so that the future is determined by the present. The linearity remains, no matter how many primitive waves are being considered at a time. If the linear differential equation is considered to be replaced by an integral equation, then the future behavior of a particular elementary particle, as seen or visualized by a certain observer, is determined by a volume integral, with a certain Green's function, over the present state of the universe, again from the point of view of the same observer. In the second step of the argument, it is noted that all of the elementary particles, to satisfy Theorems II and III, are structures which must extend over a finite volume, a volume which can in no case be less than $1/\kappa^3$. As the third step, it is concluded that an irreducible portion of the volume integral (the "causal integral") must be taken over the <u>interior</u> of the elementary particle.

Since there is no reason for the Green's function to vanish over this interior portion, it follows that the behavior of this particular elementary particle, while entirely causal, is partially self-determined. The interior region has, a priori, just as much right to share in the causal integral as any other part of the universe. The irreducible volume of the structure corresponds to the volume per wave in the Fermi sea of primitive waves, so that, a priori, there will be at least one degree of freedom associated with this volume.

An experimenter, seeking to control the motion of the elementary particle, may be able to establish the external conditions to high precision, but he cannot penetrate to the interior of the particle without destroying its identity. Any penetration would have to be by the agency of some other elementary particle; all other elementary particles are also extended in space and can bring back information dealing not with small sub-volumes within the structure but with interactions of the structure as a whole. And, since all particles are built from the same kind of waves, any such penetration would lead to exchange and reassembly, a loss of identity of the original particle.

Thus the experimenter cannot prescribe the whole of the causal integral and cannot have complete control over the behavior of the particle. No matter how far back in time the volume integration is pushed, there will always be a region which belongs to the particle and is beyond the control of the experimenter. This separation of the object from the experimenter, and allocation of degrees of freedom on each side, is concerned with actual knowledge or actual control. There is nothing to prevent the experimenter from understanding or visualizing the internal degrees of freedom, or from writing wave equations which include them. He is only prevented from knowing or controlling their quantitative values. If this were not so, if one part of the universe could be completely controlled by another part, then the picture could be reduced to an absurdity, with the whole universe under the control of a few atoms. If the universe is a functioning whole, which it seems to be, then all parts of the universe, all elementary particles, must share in that functioning. And if an electron can be shown to have an inner degree of freedom, an element of self-determination, a modicum of free will, then who is to deny a share in causality to other, larger, more complex forms of matter?

In addition to the general kind of causality, discussed in the preceding pages, there is a more restricted relationship which has also been denoted by the term "causality." The restricted causality principle is a limitation imposed upon the form that acceptable theories may take. The principle is based on the premise, from special relativity, that the propagation of an effect from one point in space to another point cannot exceed the velocity c. From this premise it is argued that the specification of a field quantity, at one point of space and time, must be independent of the specification of the same field quantity at a different point of space and time, as long as the two points of space-time are separated by a "space-like interval," so that each point is outside the "light-cone" through the other point. This restricted causality principle contains within its statement certain implicit or hidden assumptions, principally the assumption that a "field quantity" can always be localized and defined at a point in space-time, without reference to other points in the vicinity. If the very definition of a field quantity includes an integration over nearby points, then the conclusion given above does not follow from its premise.

The multiple-wave fields of the present theory are nonlocal fields falling outside the implicit assumptions of this restricted causality principle. Within the framework of the theory it is not possible for effects to be propagated at a velocity exceeding c, since all effects must be carried by primitive waves, and c is the velocity of a primitive wave. Nevertheless, a multiple-wave "observable" field quantity, such as the electric field vector, is specified by the expectation value of an operator, and the calculation of the expectation value involves an integration over a neighboring volume. The electric field is thus a nonlocal field in the sense that the value associated with one point depends upon conditions at other points in the neighborhood.

The illustration can be made more explicit. The operator for the electric vector is a double-wave dimensionless spin operator:

$$\vec{F}_e = \frac{1}{2i}\left(\tau_{1\xi}\tau_{2\eta} - \tau_{1\eta}\tau_{2\xi}\right)\left(\vec{\sigma}_1 \times \vec{\sigma}_2\right) \qquad (6)$$

The value of the electric field at a point is given by the expectation value of the operator (6) in the

Fermi sea, at that point. But the calculation of the expectation value includes an integration over the relative coordinates, the components of the relative vector $\vec{r} = \vec{r}_1 - \vec{r}_2$. The value of the electric field at one point thus depends on an integration over a surrounding volume. The electric field at a different but nearby point will also depend on a volume integration, and if the two points are very close the volumes will overlap and be almost equivalent, so that the two values for the electric field will not be independent, even though the two points are being considered at essentially the same moment in time and are thus separated in space-time by a "space-like" interval. In practice, the separation within which the two integrations become almost equivalent is determined by the shortest wavelength in the Fermi sea, which corresponds to the wavenumber κ at the top of the sea. If the separation of the two points is less than the distance (2), then field quantities computed at the two points cannot be considered independent. In delocalizing or "smearing out" field quantities such as the electric field vector, the wavelength at the top of the Fermi sea performs, once again, one of the functions of a "fundamental length."

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